Decay of Boundary-Layer Turbulence in Near Wake of a Slender Body

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A theoretical study of the decay of the fluctuating velocities in the supersonic near wake expansion of boundary-layer turbulence is presented. A model based on linear theories is utilized to predict the change in fluctuation levels and turbulent scale sizes. The effect of the compressibility of the mean flow is included, and mean properties are calculated from method of characteristics solutions. The validity of linear theory is examined, and an analysis of the decay of eddy turbulence associated with the vorticity field is presented. Results indicate that the decay rate is strongly dependent upon proper specification of the anisotropic boundary-layer turbulence. Results consistent with data for the turbulence decay in the near wake of a slender wedge are obtained when a realistic choice of the boundary-layer turbulent energy spectrum and velocity fluctuation levels is made.

Introduction

THE near wake of a slender body in hypersonic flow has received considerable attention because of its role in the formation of the turbulent far wake. Numerous theoretical and experimental studies of the structure of the near wake with a laminar boundary layer on the body exist; however, with a turbulent boundary layer this structure changes. A laminar boundary-layer flow expands rapidly into the base region and turbulent transition occurs at the neck. When the Reynolds number exceeds that required for transition, the boundary layer becomes turbulent, and the near wake region then contains the expanded boundary-layer fluctuations. The structure of the turbulence in the inner core, which grows to form the turbulent far-wake, also appears much finer than in the laminar case. To determine the effect of a turbulent boundary layer in the formation of the wake, it is first necessary to predict what happens to the boundary-layer turbulence as it passes through the rapid expansion at the vehicle corner.

The effects of a high acceleration on a turbulent supersonic shear layer were examined by Sternberg[1] and Morkovin.[2] Their results, plus the results of numerous studies of the effects of acceleration on the turbulence in wind tunnels,[3-9] lead one to conclude that the turbulent fluctuations will decay as the flow expands around the corner, with the magnitude of the decay enhanced by both the acceleration of the mean flow and the decrease in the mean density. An experimental investigation of the decay of the "turbulent remnant" of the boundary layer in the near wake of a 6° half-angle wedge at a Mach number of 4.0 has been presented by Lewis and Behrens.[10] Traversing the near wake with a hot wire anemometer, they found that the peak of the voltage fluctuations originating in the boundary-layer decay rapidly in the expansion.

Theoretical studies of the effects of flow acceleration on turbulent fluctuations have been directed toward determining wind-tunnel contraction ratios necessary to smooth out flow irregularities before the flow enters the test section. Most of this work has been restricted to small, rapid changes in an incompressible mean flow to enable linearization of the equations describing the change in fluctuation levels induced by mean flow distortion. The earliest work, by Prandtl,[3] considered the effect of a symmetrical contraction on small, steady disturbances to obtain estimates of the dependence of fluctuation energy components on wind-tunnel contraction ratio. Taylor[4] was the first to consider the effect of stream distortion on a spatially varying disturbance and he examined the effect of flow acceleration on a single wave number of turbulence. This was extended to the full turbulent spectrum for arbitrary distortions of isotropic turbulence by Batchelor and Proudman.[5] Ribner and Tucker[6] worked out the effect of the symmetrical contraction of a compressible mean flow on isotropic turbulence. The experimental results of Uberoi[7] in a low-turbulence, subsonic wind tunnel demonstrated that for large contraction ratios, there is a complete reversal of tendencies observed in small contractions and predicted by the linear theory. It was found that for large contraction ratios the velocity fluctuations in the stream direction initially decay and then increase during the latter stages of the contraction. These results prompted attempts to extend the linear theories to include nonlinear effects and the effects of dissipation. Pearson[8] presented a linearized Eulerian treatment of the effect of uniform distortion on incompressible, homogeneous turbulence, including dissipation due to viscosity, and Harlow and Romero,[9] modeling the nonlinear turbulent transport equations of incompressible, homogeneous turbulence, obtained agreement with Uberoi's experimental data.

The present paper presents a theoretical study of the decay of velocity fluctuations in the near wake expansion of boundary-layer turbulence. Since the expansion is very rapid, the linearizing assumptions made in the preceding analyses are shown to be valid. However, the assumptions of initially isotropic turbulence and an axisymmetric expansion must be abandoned, and the present analysis includes the effect of initially anisotropic turbulence on the turbulence decay rate. The linearized theory is applied to the boundary-layer mean flow streamlines as the boundary layer expands into the near wake. The effect of the compressibility of the mean flow on the fluctuation decay is included and mean properties along the streamlines are calculated from existing[11] near wake method of characteristics solutions. Calculations of the decay of boundary-layer turbulence in the near wake of a 6° wedge with $M_\infty = 4.0$ are compared with the experimental data of Lewis and Behrens.

Derivation of Equations for the Vorticity Field

The principal interest in this paper is the decay of the velocity fluctuations in the near wake expansion. Therefore, any consideration of the different types of fluctuations present in

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supersonic flow[12] is limited to determining the contribution of each of the modes of fluctuations (vorticity, sound, entropy) to the velocity fluctuation level and its decay. Kovasznay[12] found that in a moderately supersonic turbulent boundary layer the contribution of sound to the over-all mass flow fluctuations is negligible. Thus, in the boundary layer and in the expansion region it is assumed that the velocity fluctuations are determined by fluctuations in vorticity and entropy. The low turbulence level in the boundary layer, with fluctuations of the order of 5% to 10% of the mean values, allows linearization of the Navier-Stokes equations about the mean velocity as in Ref. 12. The total velocity fluctuation can be separated into a solenoidal and an irrotational fluctuation

$$\mathbf{v} = \mathbf{v}_1 + \mathbf{v}_2$$

where $\mathbf{v}_1$ is the solenoidal velocity fluctuation which, after linearization, to lowest order can only be associated with the vorticity mode; $\mathbf{v}_2$ is the potential velocity fluctuation induced by entropy "spottiness" in the boundary layer and is negligible in most practical cases.[12] The higher order contribution of the entropy mode to the solenoidal velocity fluctuations will be proportional to the stagnation temperature fluctuations. Following Sternberg,[1] we can investigate the velocity fluctuations induced by stagnation temperature fluctuations by utilizing the expression derived by Corrsin[13] for the velocity fluctuation at any point along a streamline as a function of the initial stagnation temperature fluctuation and the local Mach number. Using this expression for the present application results in induced velocity fluctuations of the order of 1% of the mean. This contribution will be neglected throughout the expansion region. This discussion and ultimate neglect of the effect of the entropy and acoustic modes on the velocity fluctuations is only reasonable for Mach numbers less than 5 (Ref. 14). There is a lack of information concerning the effects at higher Mach numbers, however, it is apparent that the interactions between the fluctuation modes become more significant at large Mach numbers, and a re-evaluation of the coupling between the entropy, sound and vorticity modes is necessary. Throughout this paper, we are primarily concerned with calculating the decay of the lowest order contribution to $\mathbf{v}_1$, i.e., the decay of the vorticity field in the expansion fan.

The starting point for previous analyses[4-7] of the effect of mean flow acceleration on the vorticity fluctuations in a turbulent fluid is Cauchy's Lagrangian equations for the transport of vorticity in an inviscid fluid.[15] Since it is difficult to visualize the way in which the nonlinear interaction terms and the dissipation mechanisms influence the transport of vorticity with this approach, an Eulerian treatment for the transport of the mean square vorticity fluctuations will be presented. The importance of the nonlinear interaction terms and dissipative effects in the expansion of the rotational shear flow is then examined and the resulting linearization yields a solution consistent with the Lagrangian approach. Consider the compressible flow of a perfect gas. Provided the turbulence level is low and the mean flow expansion is nearly isentropic, the gas can also be assumed to be barotropic. The equations for the conservation of mass and momentum with constant viscosity become[16]

$$\begin{aligned}\partial\rho/\partial t + \nabla\cdot(\rho\mathbf{U}) = 0 \\ \rho(\partial\mathbf{U}/\partial t + \mathbf{U}\cdot\nabla\mathbf{U}) = -\nabla p + \mu\nabla^2\mathbf{U} + (\mu/3)\nabla(\nabla\cdot\mathbf{U})\end{aligned} \tag{1}$$

where ρ is the density, p is the pressure, $\mathbf{U}$ is the velocity vector, and μ is the viscosity. Assuming the turbulence to be superimposed upon the mean flow, we then write in the usual manner

$$\mathbf{U} = \langle\mathbf{U}\rangle + \mathbf{u}' \qquad p = \langle p\rangle + p' \qquad \rho = \langle\rho\rangle + \rho' \tag{2}$$

where the bracket denotes the suitably averaged mean flow property and $\mathbf{u}'$, p' and ρ' are the turbulent fluctuations in velocity, pressure, and density, respectively. Substituting Eq. (2) into Eq. (1), averaging, and assuming $\rho'/\langle\rho\rangle \ll 1$, the continuity and momentum equations for the mean flow become

$$\begin{aligned}&\nabla\cdot\langle\rho\rangle\langle\mathbf{U}\rangle = 0 \\ &\langle\rho\rangle(\partial\langle\mathbf{U}\rangle/\partial t + \langle\mathbf{U}\rangle\cdot\nabla\langle\mathbf{U}\rangle) = -\nabla\langle p\rangle + \mu\nabla^2\langle\mathbf{U}\rangle + \\ &\qquad (\mu/3)\nabla(\nabla\cdot\langle\mathbf{U}\rangle) - \langle\rho\rangle\langle\mathbf{u}'\cdot\nabla\mathbf{u}'\rangle\end{aligned} \tag{3}$$

The neglect of the density fluctuations in Eq. (3) implies that the compressibility of the turbulence is negligible. Hinze[16] shows that $\rho'/\langle\rho\rangle$ is of the order of the Mach number of the turbulence, $u'^2/\langle c\rangle^2$, where u' denotes the velocity fluctuation and $\langle c\rangle$ is the sound speed based on the mean flow properties. It can be shown from existing measurements and mixing length arguments that even for high Mach number flows, the level of turbulence in the boundary layer is such that the turbulence Mach number is much less than one. It is also assumed that in the expansion process the temperature does not drop faster than the fluctuation level to enhance substantially the turbulence Mach number. Substracting Eq. (3) from Eq. (1) and dropping the primes for convenience, we obtain the relevant equations for the velocity fluctuations

$$\nabla\cdot(\langle\rho\rangle\mathbf{u}) = 0 \tag{4a}$$

$$\begin{aligned}\partial\mathbf{u}/\partial t + \mathbf{u}\cdot\nabla\langle\mathbf{U}\rangle + \langle\mathbf{U}\rangle\cdot\nabla\mathbf{u} + \mathbf{u}\cdot\nabla\mathbf{u} = -(1/\langle\rho\rangle)\nabla p + \\ \nu\nabla^2\mathbf{u} + \langle\mathbf{u}\cdot\nabla\mathbf{u}\rangle\end{aligned} \tag{4b}$$

To obtain the differential equation describing the vorticity field, we take the curl of Eq. (4b). Performing the necessary vector operations and using the continuity equations for both the mean and fluctuating velocities, we can write the equation for the conservation of vorticity fluctuations as

$$\begin{aligned}&\frac{\partial(\boldsymbol{\omega}/\langle\rho\rangle)}{\partial t} + \langle\mathbf{U}\rangle\cdot\nabla\left(\frac{\boldsymbol{\omega}}{\langle\rho\rangle}\right) = \left(\frac{\boldsymbol{\omega}}{\langle\rho\rangle}\right)\cdot\nabla\langle\mathbf{U}\rangle + \\ &\quad \frac{(\boldsymbol{\Omega}+\boldsymbol{\omega})}{\langle\rho\rangle}\cdot\nabla\mathbf{u} - \mathbf{u}\cdot\nabla\frac{(\boldsymbol{\Omega}+\boldsymbol{\omega})}{\langle\rho\rangle} + \\ &\qquad \frac{\nu}{\langle\rho\rangle}\nabla^2\boldsymbol{\omega} - \left\langle\left(\frac{\boldsymbol{\omega}}{\langle\rho\rangle}\right)\cdot\nabla\mathbf{u}\right\rangle + \left\langle\mathbf{u}\cdot\nabla\left(\frac{\boldsymbol{\omega}}{\langle\rho\rangle}\right)\right\rangle\end{aligned} \tag{5}$$

where $\boldsymbol{\omega}$ denotes the vorticity fluctuation vector and $\boldsymbol{\Omega}$ is the vorticity vector associated wth the mean flow. The additional steps necessary to get to an expression for the mean square vorticity fluctuations can best be seen by rewriting Eq. (15) in Cartesian tensor notation to yield

$$\begin{aligned}&\frac{\partial}{\partial t}\left(\frac{\omega_i}{\langle\rho\rangle}\right) + \langle U_k\rangle\frac{\partial}{\partial x_k}\left(\frac{\omega_i}{\langle\rho\rangle}\right) = \left(\frac{\omega_k}{\langle\rho\rangle}\right)\frac{\partial\langle U_i\rangle}{\partial x_k} + \\ &\quad \frac{(\Omega_k+\omega_k)}{\langle\rho\rangle}\frac{\partial u_i}{\partial x_k} - u_k\frac{\partial}{\partial x_k}\frac{(\Omega_i+\omega_i)}{\langle\rho\rangle} + \\ &\qquad \frac{\nu}{\langle\rho\rangle}\frac{\partial^2\omega_i}{\partial x_l\partial x_l} - \left\langle\frac{\omega_k}{\langle\rho\rangle}\frac{\partial u_i}{\partial x_k}\right\rangle + \left\langle u_k\frac{\partial}{\partial x_k}\left(\frac{\omega_i}{\langle\rho\rangle}\right)\right\rangle\end{aligned} \tag{6}$$

Following standard techniques[16] we multiply Eq. (6) by $\omega_j/\langle\rho\rangle$, multiply Eq. (6) (written for $\omega_j/\langle\rho\rangle$) by $\omega_i/\langle\rho\rangle$, add, manipulate, set $j = i$, and average to obtain

$$\begin{aligned}&\frac{1}{2}\frac{\partial}{\partial t}\left(\frac{\langle\omega_i^2\rangle}{\langle\rho\rangle^2}\right) + \frac{\langle U_k\rangle}{2}\frac{\partial}{\partial x_k}\left(\frac{\langle\omega_i^2\rangle}{\langle\rho\rangle^2}\right) + \\ &\left\langle\frac{u_k}{2}\frac{\partial}{\partial x_k}\left(\frac{\omega_i^2}{\langle\rho\rangle^2}\right)\right\rangle = \frac{\langle\omega_i\omega_k\rangle}{\langle\rho\rangle^2}\frac{\partial\langle U_i\rangle}{\partial x_k} + \\ &\left\langle\frac{\omega_i\omega_k}{\langle\rho\rangle^2}\frac{\partial u_i}{\partial x_k}\right\rangle + \left\langle\frac{\omega_i\Omega_k}{\langle\rho\rangle^2}\frac{\partial u_i}{\partial x_k}\right\rangle - \frac{\langle\omega_i u_k\rangle}{\langle\rho\rangle}\times \\ &\qquad \frac{\partial}{\partial x_k}\left(\frac{\Omega_i}{\langle\rho\rangle}\right) + \frac{\nu}{2\langle\rho\rangle^2}\frac{\partial\langle\omega_i^2\rangle}{\partial x_l\partial x_l} - \frac{\nu}{\langle\rho\rangle^2}\left\langle\left(\frac{\partial\omega_i}{\partial x_l}\right)^2\right\rangle\end{aligned} \tag{7}$$

where summation is over k and l but not i. Equation (7) describes the balance of the mean square vorticity fluctuation for one component of vorticity fluctuations in the fluid. It is akin to the turbulence-energy equation which describes the balance of energy per unit mass for each component of the velocity fluctuations.[16-17] The first two terms on the left side of Eq. (7) give the total change in the mean square vorticity fluctuation, while the third term may be interpreted as the convective diffusion by turbulence of the mean square vorticity

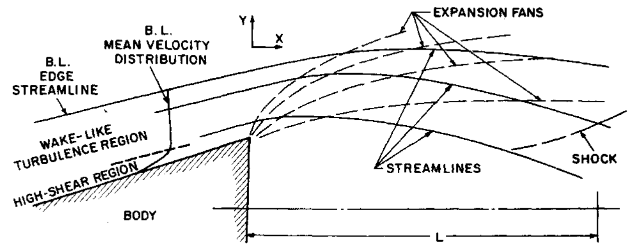

Fig. 1 Sketch of the near wake expansion region.

fluctuation. The first two terms on the right side of Eq. (7) are considered to be the work done per unit mass by a stress akin to the Reynolds stress acting on the mean flow and the turbulence, while the third and fourth terms may be interpreted as work terms associated with the rotationality of the mean flow. The viscous terms are usually associated with the work done by the viscous stresses of the turbulence and the dissipation per unit mass by the turbulent motion. Equation (7) in its present form is intractable; however, we now specialize this description of the vorticity field to the case of a supersonic turbulent boundary layer flow undergoing a rapid expansion.

Vorticity Field in the Near Wake Expansion

A sketch of the flow configuration of interest is shown in Fig. 1. A typical turbulent boundary-layer mean velocity profile is indicated and the boundary layer has been divided into two regions. The viscous sublayer and wall-dominated turbulence layer have been combined and labeled a high shear region, while the large outer turbulent layer where the turbulence is nearly homogeneous is usually called the wakelike turbulence region. Turbulence is generated primarily in the high shear region and diffuses into the outer region where turbulence production is nearly balanced by dissipation creating a nearly homogeneous field.[18] The intermittency associated with the outer edge of the boundary layer is neglected and in this paper we restrict ourselves to a consideration of the expansion of the wake-like region of the boundary layer. The length of this expansion region has been denoted by L.

To determine if linearization of Eq. (7) is physically justifiable for the present problem, the simplifications which arise from the rapid expansion of small scale turbulence when considering the nonlinear terms and the dissipation terms in Eq. (7) must be examined. First, the wakelike region of the turbulent boundary layer is nearly homogeneous, and we can neglect the convective diffusion of the mean square vorticity fluctuations by the velocity fluctuations in the boundary layer and the subsequent expansion. Therefore, the third term on the left side of Eq. (7) is dropped. The work done per unit mass by a virtual turbulent stress acting on the turbulence can be neglected if

$$\partial u_i/\partial x_k \ll \partial\langle U_i\rangle/\partial x_k \tag{8}$$

Since $\partial u_i/\partial x_k = O(u/\Lambda)$ where Λ is the scale of the turbulence, and $\partial\langle U_i\rangle/\partial x_k = O(\langle U\rangle/L)$, this condition becomes

$$u/\langle U\rangle \ll \Lambda/L \tag{9}$$

The scale of the turbulence will be some fraction of the boundary-layer thickness at the end of the body. Thus, we may write

$$\Lambda = k_1\delta \sim k_1 l_B/Re^{1/5} \tag{10}$$

where k_1 is a constant, δ is the boundary-layer thickness at the end of the body, l_B is the length of the body and Re is the Reynolds number based on body length. Assuming the length of the expansion region to be of the order of the base diameter and using the relation $\theta l_B \approx d/2$ for a slender body, where θ is the cone or wedge half-angle and d is the base diameter, we can write the condition Eq. (9) as

$$k_1 k_2/\theta^{4/5} Re_d^{\,1/5} \gg 1 \tag{11}$$

where $k_2 = \langle U\rangle/u$ and Re_d is the Reynolds number based on the base diameter. If Eq. (11) is satisfied, we can then neglect the second term on the right side of Eq. (7). The work terms associated with the rotationality of the mean flow receive contributions only from the mean flow vorticity vector in the z direction and the magnitude of this vector $\sim\partial\langle U\rangle/\partial y$. The wake-like region of the boundary layer is the region of minimum mean shear in the boundary layer and therefore we can neglect the nonlinear effect of the rotationality of the mean flow on the turbulence in the expansion. The first of the two viscous terms is also dropped since the work done by the viscous stresses of the turbulence is a diffusionlike term[17] and is neglected. The characteristic time for the dissipation per unit mass of the turbulence in a high Reynolds number flow can be written[17]

$$\tau_D \sim \Lambda/u \tag{12}$$

Therefore, the length scale over which dissipation becomes important is

$$l_D \sim \Lambda\langle U\rangle/u \tag{13}$$

Equations (9) and (13) imply that $L \ll l_D$; thus, the condition expressed by Eq. (11) also becomes the criterion for the neglect of turbulent dissipation in the expansion.

With the linearization condition Eq. (11) satisfied, Eq. (7) becomes

$$\frac{\partial}{\partial t}\left(\frac{\langle\omega_i^2\rangle}{\langle\rho\rangle^2}\right)+\langle U_k\rangle\frac{\partial}{\partial x_k}\left(\frac{\langle\omega_i^2\rangle}{\langle\rho\rangle^2}\right)=\frac{2\langle\omega_i\omega_k\rangle}{\langle\rho\rangle^2}\frac{\partial\langle U_i\rangle}{\partial x_k} \tag{14}$$

If the body is a slender cone or wedge, the expansion process is two-dimensional. Also, if the body is sufficiently slender, the mean-flow streamlines will lie approximately along the x direction (indicated in Fig. 1). Therefore, in the expansion $\langle U\rangle \gg \langle V\rangle$ and $\langle W\rangle = 0$, where $\langle U\rangle$, $\langle V\rangle$ and $\langle W\rangle$ are the mean flow velocities in the x, y and z directions, respectively. In addition [provided $\langle\omega_i\omega_j\rangle = O(1)$], the neglect of the effect of the rotationality of the mean flow on the turbulence implies that

$$\langle\omega_x\omega_y\rangle\,\partial\langle U\rangle/\partial y \ll \langle\omega_x^2\rangle\,\partial\langle U\rangle/\partial x \tag{15}$$

and

$$\langle\omega_x\omega_y\rangle\,\partial\langle V\rangle/\partial x \ll \langle\omega_y^2\rangle\,\partial\langle V\rangle/\partial y$$

Utilizing Eq. (15) and the condition $\langle U\rangle \gg \langle V\rangle$, and specializing to steady flow, we obtain for the three components of the mean square vorticity fluctuations

$$\begin{aligned}\langle U\rangle\,\partial(\langle\omega_x^2\rangle/\langle\rho\rangle^2)/\partial x &= (2\langle\omega_x^2\rangle/\langle\rho\rangle^2)\,\partial\langle U\rangle/\partial x\\ \langle U\rangle\,\partial(\langle\omega_y^2\rangle/\langle\rho\rangle^2)/\partial x &= (2\langle\omega_y^2\rangle/\langle\rho\rangle^2)\,\partial\langle V\rangle/\partial y\\ \langle U\rangle\,\partial(\langle\omega_z^2\rangle/\langle\rho\rangle^2)/\partial x &= 0\end{aligned} \tag{16}$$

After using the continuity equation for the mean flow, Eqs. (16) are immediately integrable and yield

$$\langle\omega_x^2\rangle/(\langle\rho\rangle\langle U\rangle)^2 = \text{const} \tag{17}$$

$$\langle\omega_y^2\rangle\langle U\rangle^2 = \text{const} \tag{18}$$

$$\langle\omega_z^2\rangle/\langle\rho\rangle^2 = \text{const} \tag{19}$$

along the streamlines in the expansion region.

Simplified Description of Change in Scale Sizes and Fluctuation Decay

With the linearizing assumptions just discussed incorporated into a model of the decaying turbulence, the distortion of a turbulent "eddy" or a single vortex as it traverses an expansion fan can be described rather simply, as is illustrated schematically in Fig. 2. Consider the turbulent "eddy" or vortex to be contained within a streamtube which passes through the expansion fan. The vortex is assumed to be initially circular of length scale Λ. Large values of Λ would correspond to turbulence scales at the low wave number end of a turbulence spectrum, while small Λ would correspond to small-scale eddies at the high wave number end of the spectrum. With subscript zero denoting conditions before the expansion and subscript one denoting conditions after the expansion, the vortex is seen to "expand" with the change in area of the streamtube.

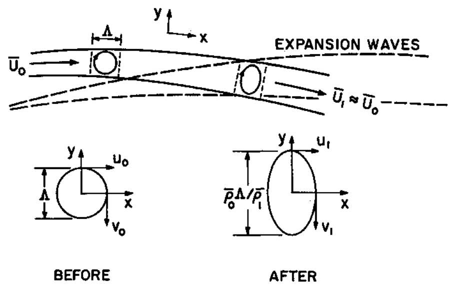

Fig. 2 Eddy or single vortex distortion in an expansion fan.

Since the mean flow is hypersonic, the expansion waves are at a slight angle to the streamline direction and most of the expansion takes place in the y direction; the mean velocity does not change significantly and $\langle U_1\rangle \approx \langle U_0\rangle$. With such a directed expansion the vortex will not distort in the x direction, but will distort in the y direction by the amount necessary to satisfy conservation of mass within the streamtube. Even if the turbulence were isotropic before the expansion, the directed stretching in the expansion would produce highly anisotropic turbulence. The streamtube undergoes a two-dimensional expansion (no distortion in z), and the circular vortex becomes elliptical with the major axis in the y direction equal to $(\langle\rho_0\rangle/\langle\rho_1\rangle)\Lambda$ and the minor axis in the x direction equal to Λ (Fig. 2).

From this simple kinematical description of the distortion of a single wave number of turbulence in the expansion, it is also possible to obtain an estimate of the decay of the velocity fluctuations by assuming that the velocity distribution within this vortex is sinusoidal.[4] With the axis of the vortex in the z direction, the fluctuation velocities can be written

$$u_0 = A_0 \cos(\pi x_0/\Lambda)\sin(\pi y_0/\Lambda); \quad v_0 = B_0 \sin(\pi x_0/\Lambda)\cos(\pi y_0/\Lambda) \tag{20}$$

where u_0 and v_0 are the velocity fluctuations in the x and y directions and the coordinate system is now centered at the vortex center and moves with the mean velocity. From the definition of the vorticity, ω_{z_0} can be found using Eq. (20), and Eq. (19) for a single vortex filament predicts

$$\omega_{z_1} = (\langle\rho_1\rangle/\langle\rho_0\rangle)\,\omega_{z_0} \tag{21}$$

Carrying out the indicated differentiation to obtain the vorticity, and utilizing Eq. (21), we obtain for the vorticity after traversing the fan

$$\omega_{z_1} = (\langle\rho_1\rangle/\langle\rho_0\rangle)(B_0 - A_0)(\pi/\Lambda)\cos(\pi x_1/\Lambda)\times \cos(\langle\rho_1\rangle\pi y_1/\langle\rho_0\rangle\Lambda) \tag{22}$$

where the hypersonic approximation that $\langle U_1\rangle \approx \langle U_0\rangle$ has been used and x_1 and y_1 are the coordinates stretched after the expansion such that $x_1 \approx x_0$ and $y_1 \approx (\langle\rho_0\rangle/\langle\rho_1\rangle)y_0$. The velocity fluctuations after expansion can be calculated by solving the system of differential equations defining the vorticity and the incompressibility of the turbulence, i.e.,

$$\omega_{z_1} = \partial u_1/\partial y_1 - \partial v_1/\partial x_1; \quad \partial u_1/\partial x_1 + \partial v_1/\partial y_1 = 0 \tag{23}$$

Solving Eq. (23) with Eq. (22) by introducing a stream function, we can obtain the velocity fluctuations after the expansion as a function of the fluctuations before and the mean density change. Defining an average as being a volumetric average over the region occupied by the vortex, we then obtain for the change in the mean square velocity fluctuations

$$\frac{\langle u_1^2\rangle}{\langle u_0^2\rangle} = \frac{4(\langle\rho_1\rangle/\langle\rho_0\rangle)^4}{[1+(\langle\rho_1\rangle/\langle\rho_0\rangle)]^2}; \quad \frac{\langle v_1^2\rangle}{\langle v_0^2\rangle} = \frac{4(\langle\rho_1\rangle/\langle\rho_0\rangle)^2}{[1+(\langle\rho_1\rangle/\langle\rho_0\rangle)^2]^2} \tag{24}$$

Equation (24) represents the change in the mean square velocity of a single vortex with its axis oriented in the z direction. The results are as expected since the directed expansion would slow down the velocity component associated with the larger eddy scale more than the velocity component associated with the minor x axis of the ellipse.

Full Spectrum Solution for Velocity Fluctuations

Equation (24) represents the decay of the velocity fluctuations from a single component of vorticity corresponding to one "eddy" size or wave number of the turbulence. Before we can apply the results of the second section to obtain the full spectrum solution, it is necessary to impose an additional restriction on the turbulence in the expansion, that is

$$\Lambda \ll L \tag{25}$$

From Eq. (10) and the assumption that the length of the expansion region is of order of the base diameter d, where $d \approx 2\theta l_B$, the condition Eq. (25) becomes

$$k_1/(\theta^{4/5} Re_d^{\,1/5}) \ll 1 \tag{26}$$

Equation (25) implies that the turbulence in the entire expansion process remains locally homogeneous, i.e., over the scale of the turbulence the effect of the mean flow gradients in x are of higher order. If, instead of deriving a differential equation for the dynamics of the one point vorticity correlation as was done in the second section, the dynamic equation for the two-point vorticity correlation $\langle\omega_{i_A}\omega_{i_B}\rangle$, where A and B are two points separated by a distance $r \lesssim \Lambda$, is derived, then Eq. (14) becomes (for steady flow)

$$\frac{\langle U_k\rangle_A + \langle U_k\rangle_B}{2}\left[\left(\frac{\partial}{\partial x_k}\right)_{AB}\left\langle\left(\frac{\omega_i}{\langle\rho\rangle}\right)_A\left(\frac{\omega_i}{\langle\rho\rangle}\right)_B\right\rangle\right] + [\langle U_k\rangle_B - \langle U_k\rangle_A]\frac{\partial}{\partial r_k}\left\langle\left(\frac{\omega_i}{\langle\rho\rangle}\right)_A\left(\frac{\omega_i}{\langle\rho\rangle}\right)_B\right\rangle = \left\langle\left(\frac{\omega_i}{\langle\rho\rangle}\right)_B\left(\frac{\omega_k}{\langle\rho\rangle}\right)_A\right\rangle\left(\frac{\partial\langle U_i\rangle_A}{\partial x_{k_A}}\right) + \left\langle\left(\frac{\omega_i}{\langle\rho\rangle}\right)_A\left(\frac{\omega_k}{\langle\rho\rangle}\right)_B\right\rangle\times\left(\frac{\partial\langle U_i\rangle_B}{\partial x_k}\right)_B \tag{27}$$

where $x_{k_{AB}}$ is the point miday between A and B. Under the conditions (25) and (26)

$$\begin{aligned}\langle U_k\rangle_B &= \langle U_k\rangle_A + O(\Lambda/L);\\ \partial\langle U_i\rangle_B/\partial x_{k_B} &= \partial\langle U_i\rangle_A/\partial x_{k_A} + O(\Lambda/L)\\ \langle\rho\rangle_B &= \langle\rho\rangle_A + O(\Lambda/L)\end{aligned} \tag{28}$$

Therefore, to lowest order in Λ/L, Eq. (27) becomes

$$\langle U_k\rangle\,\partial(\langle\omega_{i_A}\omega_{i_B}\rangle/\langle\rho\rangle^2)/\partial x_k = 2(\langle\omega_{i_A}\omega_{k_B}\rangle/\langle\rho\rangle^2)\,\partial\langle U_i\rangle/\partial x_k \tag{29}$$

Following the steps that led to Eqs. (17–19), we obtain the analogous results from Eq. (29) for the change in the two-point vorticity correlations along the streamlines

$$\langle\omega_{x_A}\omega_{x_B}\rangle/(\langle\rho\rangle\langle U\rangle)^2 = \text{const} \tag{30}$$

$$\langle\omega_{y_A}\omega_{y_B}\rangle\langle U\rangle^2 = \text{const} \tag{31}$$

$$\langle\omega_{z_A}\omega_{z_B}\rangle/\langle\rho\rangle^2 = \text{const} \tag{32}$$

It is now possible to transform to spectrum space and proceed to calculate the mean square velocity fluctuations as in Refs. 5 and 6. After defining

$$\begin{aligned}\Phi_{ij} &= \int\langle u_i(\mathbf{x})u_j(\mathbf{x}+\mathbf{r})\rangle e^{-i\mathbf{k}\cdot\mathbf{r}}\,d\mathbf{r}\\ \Omega_{ij} &= \int\langle\omega_i(\mathbf{x})\omega_j(\mathbf{x}+\mathbf{r})\rangle e^{-i\mathbf{k}\cdot\mathbf{r}}\,d\mathbf{r}\end{aligned} \tag{33}$$

we Fourier transform Eqs. (30–32) written for any two points (subscripted zero and one) along a streamline, and utilizing the conditions Eqs. (25) and (26) so that the $\mathbf{r}$ integration is significant only over a scale length Λ much less than the distance between points zero and one, we obtain

$$(\Omega_{xx})_1 = (\langle\rho_1\rangle\langle U_1\rangle/\langle\rho_0\rangle\langle U_0\rangle)^2(\Omega_{xx})_0 \tag{34}$$

$$(\Omega_{yy})_1 = (\langle U_0\rangle/\langle U_1\rangle)^2(\Omega_{yy})_0 \tag{35}$$

$$(\Omega_{zz})_1 = (\langle\rho_1\rangle/\langle\rho_0\rangle)^2(\Omega_{zz})_0 \tag{36}$$

To lowest order in the turbulence Mach number the turbulence is incompressible so the result derived by Batchelor[19] for the energy spectrum tensor in terms of the vorticity spectrum tensor is valid. Thus

$$k^2\Phi_{ij} = \left(\frac{\delta_{ik}k^2 - k_i k_j}{k^2}\right)\Omega_{ll} - \Omega_{ji} \tag{37}$$

Using Eqs. (34–36), we can now solve for the energy spectrum of the turbulence at any point along a streamline in the expansion as a function of the initial spectrum in the boundary layer and the change in mean flow properties during the expansion. The result for $(\Phi_{xx})_1$ in the hypersonic limit ($\langle U_1\rangle \approx \langle U_0\rangle$) becomes

$$\kappa^2(\Phi_{xx})_1 = \varepsilon^{-1}\Bigg\{(k_x{}^2+k_y{}^2+\varepsilon^2 k_z{}^2)(\Phi_{xx})_0 + \left[\frac{(k_y{}^2+\varepsilon^2 k_z{}^2)(k_x{}^2+k_y{}^2(2-\varepsilon^2)+k_z{}^2)}{\varepsilon^2(k_x{}^2+k_z{}^2)+k_y{}^2} - (k_y{}^2+k_z{}^2)\right]\times (\Phi_{yy})_0 + (\varepsilon^2-1)k_z{}^2(\Phi_{zz})_0\Bigg\} \quad (38)$$

where $\varepsilon^2 = (\langle\rho_0\rangle/\langle\rho_1\rangle)^2$, $\kappa^2 = k_x{}^2+(\langle\rho_1\rangle k_y/\langle\rho_0\rangle)^2+k_z{}^2$ and k_x, k_y and k_z are the wave number components in the x, y and z directions, respectively. Similar results follow for the $(\Phi_{yy})_1$ and $(\Phi_{zz})_1$ components of the spectrum tensor but have been omitted for brevity. From the definitions, Eq. (33),

$$\langle u_1{}^2\rangle = \int(\Phi_{xx})_1\, d\mathbf{k};\quad \langle v_1{}^2\rangle = \int(\Phi_{yy})_1\, d\mathbf{k};\quad \langle w_1{}^2\rangle = \int(\Phi_{zz})_1\, d\mathbf{k} \quad (39)$$

where $\langle u_1{}^2\rangle$, $\langle v_1{}^2\rangle$ and $\langle w_1{}^2\rangle$ are the mean square velocity fluctuations in the x, y, and z directions at point one, and solutions for the mean square velocity fluctuations along the streamlines can be found immediately by integrating Eq. (38) over all wave numbers once the boundary-layer turbulence spectrum is known.

Numerical Results

Since the only existing data for the decay of boundary-layer turbulence in the near wake expansion region of a slender body is that of Lewis and Behrens,[10] we will use their flow conditions for an illustrative example. Lewis and Behrens made fluctuation measurements in the near wake of a 6° half-angle wedge immersed in a Mach number 4.0 freestream. First, to determine whether the linearized treatment of the expansion effect on the turbulence is valid for their flow conditions we utilize Eq. (11). Measurements of the decay of the "turbulent remnant" of the boundary layer were made for both tripped and untripped boundary layers with $Re_d = 3.0\times 10^5$. Taking the scale of the turbulence to be one quarter of the boundary-layer thickness and the fluctuation velocities to be initially 5% of the mean, we obtain approximately 3.5 for the parameter in Eq. (11) at the end of the boundary layer. As the flow begins to expand, the velocity fluctuations decay and the scale of the turbulence increases (Fig. 2) making this parameter larger. Thus, we conclude that the nonlinear terms and the effects of turbulent dissipation are negligible. The decay of the velocity fluctuations is attributed solely to the contracting of a distribution of fluctuating vortex filaments by the expanding mean flow.

In Fig. 3, the near wake flowfield and fluctuation intensity survey for a tripped turbulent boundary layer, presented by

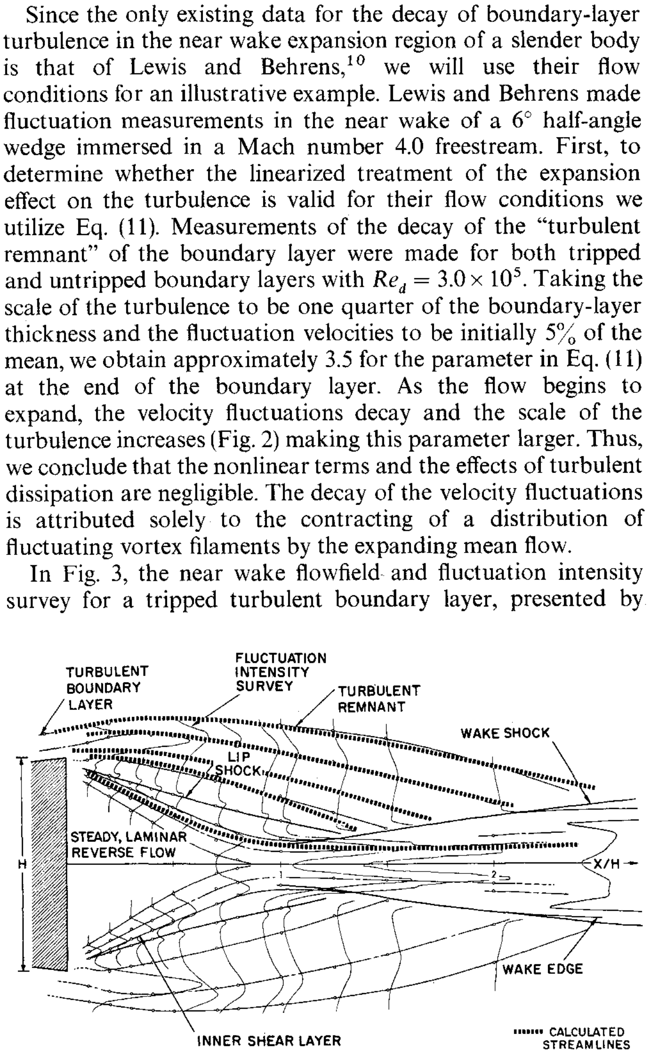

Fig. 3 Near wake flowfield of slender wedge with tripped boundary layer from Lewis and Behrens,[10] and calculated mean flow streamlines.

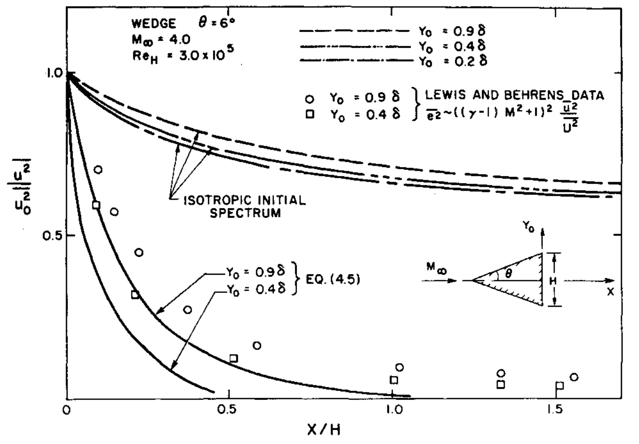

Fig. 4 Results for axial mean square velocity fluctuation decay with an isotropic boundary-layer turbulence spectrum; single vortex model prediction, and data of Lewis and Behrens interpreted in terms of velocity fluctuations.

Lewis and Behrens as Fig. 7 in Ref. 10, is shown. Superimposed are calculated results for the mean flow streamline pattern obtained by utilizing existing[11] method of characteristics solutions for the mean flowfield in the near wake. From this computer solution the values of the mean velocity, temperature, pressure, density, Mach number, and flow angle along specified streamlines are also obtained. Before Eqs. (38) and (39) can be used to calculate the velocity fluctuation levels, it is necessary to determine the spectrum of the turbulence in the boundary layer. There exists very little data concerning the spectrum of supersonic boundary-layer turbulence; however, certain qualitative observations can be made from the existing fluctuation data. The turbulence is highly anisotropic in the high shear region of the boundary layer with fluctuations generated primarily in the mean flow direction.[16] As the turbulence diffuses away from the wall, the degree of anisotropy decreases as mutual exchange of energy between fluctuation components drives the turbulence toward isotropy. Since no definitive form for the anisotropic spectrum in the boundary layer exists, we will determine the effects of this anisotropy on the solution in the expansion region by first comparing the solutions obtained assuming an isotropic initial spectrum with the existing data. Therefore, we assume an initial energy spectrum of the form[6]

$$(\Phi_{ij})_0 = G(k)(k^2\delta_{ij} - k_i k_j) \quad (40)$$

where $k^2 = k_x{}^2+k_y{}^2+k_z{}^2$ and $G(k)$ is an arbitrary function of wave number. Substituting Eq. (40) into Eq. (38) and performing the wave number integrations indicated by Eq. (39), we can obtain the mean square velocity fluctuations along streamlines throughout the expansion region once the values of the mean density along indicated streamlines are inserted into Eq. (38). These results for the mean square velocity fluctuations in the x direction are presented in Fig. 4. To be consistent with the notation of Ref. 10, H has been substituted for d as the base diameter. Results are presented along particular streamlines with the initial y location of the streamline at the end of the boundary layer denoted by a distance y_0 equal to a fraction of the boundary layer thickness δ above the wall. As expected, the streamlines originating closer to the wall with a lower initial Mach number will expand more, enhancing the fluctuation decay. In Fig. 9 of Ref. 10 the decay of the peak intensity of the voltage fluctuations in the turbulent remnant of the boundary layer is presented. To interpret this data and obtain the decay of the mean square velocity fluctuations in the axial direction, the analysis for the interpretation of hot-wire measurements in supersonic flow made by Kovasznay,[20] along with the results for the calculated voltage sensitivity coefficients presented in Ref. 21, were utilized.† The results of this data reduction are also

† The author would like to thank J. E. Lewis of TRW Systems for his suggestions regarding the interpretation of his data.

presented in Fig. 4 along with the relation which transforms voltage fluctions $\overline{e^2}$ to the velocity fluctions.‡ The difference in the reduced peak intensity data for two different streamlines is due to the different mean flow Mach numbers. It is apparent from Fig. 4 that with an initially isotropic spectrum the theory predicts a much slower decay of the velocity fluctuations than is actually observed. Results predicted by Eq. (24) with the simple single vortex model of the decaying turbulence are also shown and they predict a faster decay than is observed. However, this model gives an indication of the effect of the anisotropy of the initial turbulence spectrum. A single vortex in the z direction can be considered to be the limit of a distribution of vorticity fluctuations in three directions whose strongest components are in the z direction. This is typical of the boundary-layer turbulence close to the wall. As an attempt to model an anisotropic spectrum and determine the effects of the boundary-layer spectrum on the solution in the expansion region, we assume a form for the initial three-dimensional energy spectrum that contains the proper number of parameters necessary to specify any degree of anisotropy. A modified Gaussian three-dimensional energy spectrum of the form

$$(\Phi_{ij})_0 = [\langle q_0{}^2\rangle \Lambda_x\Lambda_y\Lambda_z/\{32\pi^{3/2}(\Lambda_x{}^2+\Lambda_y{}^2+\Lambda_z{}^2)\}]\Lambda_i\Lambda_j \times \exp\{-\tfrac{1}{4}(k_x{}^2\Lambda_x{}^2+k_y{}^2\Lambda_y{}^2+k_z{}^2\Lambda_z{}^2)\}\cdot \{(k_x{}^2\Lambda_x{}^2+k_y{}^2\Lambda_y{}^2+k_z{}^2\Lambda_z{}^2)\delta_{ij}-k_i\Lambda_i k_j\Lambda_j\} \quad (41)$$

can be derived by assuming a Gaussian form and using continuity to obtain the proper normalization. $\langle q_0{}^2\rangle$ is the total mean square velocity fluctuation at a point and Λ_x, Λ_y, and Λ_z are the integral scales defined such that

$$\begin{aligned}\Lambda_x &= \frac{2}{(\pi)^{1/2}}\int_0^\infty \frac{\langle u_x u_x(r_x,0,0)\rangle\, dr_x}{\langle u_0{}^2\rangle}\\ \Lambda_y &= \frac{2}{(\pi)^{1/2}}\int_0^\infty \frac{\langle u_y u_y(0,r_y,0)\rangle\, dr_y}{\langle v_0{}^2\rangle}\\ \Lambda_z &= \frac{2}{(\pi)^{1/2}}\int_0^\infty \frac{\langle u_z u_z(0,0,r_z)\rangle\, dr_z}{\langle w_0{}^2\rangle}\end{aligned} \quad (42)$$

where $\langle u_x u_x(r_x,r_y,r_z)\rangle$, $\langle u_y u_y(r_x,r_y,r_z)\rangle$ and $\langle u_z u_z(r_x,r_y,r_z)\rangle$ are two-point velocity correlations for x, y, and z velocity fluctuations, respectively, and r_x, r_y, and r_z are the distance components between the points. This form of the turbulent energy spectrum is convenient to work with and also gives adequate agreement with the one-dimensional longitudinal spectrum data obtained by Kistler[22] in a supersonic turbulent boundary layer when integrated over all k_y and k_z. The degree of anisotropy of the turbulence is specified by the components of the mean square velocity fluctuations in the boundary layer, $\langle u_0{}^2\rangle$, $\langle v_0{}^2\rangle$, and $\langle w_0{}^2\rangle$, which sum to form $\langle q_0{}^2\rangle$ and can also be related to the integral scales, Λ_x, Λ_y, and Λ_z. When $\langle u_0{}^2\rangle = \langle v_0{}^2\rangle = \langle w_0{}^2\rangle, \Lambda_x = \Lambda_y = \Lambda_z$ and Eq. (41) reduces to an isotropic initial energy spectrum [Eq. (40)]. By taking the inverse Fourier transform of Eq. (41) to determine the velocity correlations, we can obtain the relationship between the mean square velocity fluctuations and the integral scales. The result is

$$\langle u_0{}^2\rangle/\langle v_0{}^2\rangle = \Lambda_x{}^2/\Lambda_y{}^2; \quad \langle u_0{}^2\rangle/\langle w_0{}^2\rangle = \Lambda_x{}^2/\Lambda_z{}^2 \quad (43)$$

Since there exists no measurements of all three components of the mean square velocity fluctuations in a supersonic turbulent boundary layer, the subsonic data of Klebanoff[23] will be used to specify the velocity fluctuations as a function of distance from the wall. In addition, there are no measurements of the integral scales Λ_x, Λ_y, and Λ_z in a supersonic turbulent boundary layer. However, contours of the three components of the longitudinal velocity correlations defined as

$$\begin{aligned}(R_{uu})_x &= \langle u_x(x_0,y_0,z_0)u_x(x_0+r_x,y_0,z_0)\rangle/\langle u_0{}^2\rangle\\ (R_{uu})_y &= \langle u_x(x_0,y_0,z_0)u_x(x_0,y_0+r_y,z_0)\rangle/\langle u_0{}^2\rangle\\ (R_{uu})_z &= \langle u_x(x_0,y_0,z_0)u_x(x_0,y_0,z_0+r_z)\rangle/\langle u_0{}^2\rangle\end{aligned} \quad (44)$$

where u_x is the velocity fluctuation in the x direction, have been obtained in a subsonic turbulent boundary layer.[24,25] Using Eqs. (41, 42, and 44), along with the definition of the turbulence spectrum, we can define the integral scales for the longitudinal correlations in terms of Λ_x, Λ_y, and Λ_z as

$$L_x = \int_0^\infty (R_{uu})_x\, dr_x = \frac{(\pi)^{1/2}}{2}\Lambda_x; \quad L_y = \int_0^\infty (R_{uu})_y\, dr_y = \frac{(\pi)^{1/2}}{4}\Lambda_y$$
$$L_z = \int_0^\infty (R_{uu})_z\, dr_z = \frac{(\pi)^{1/2}}{4}\Lambda_z \quad (45)$$

Existing measurements[26,27] indicate that the eddies in the wake-like region of the boundary layer are elongated in the streamline direction such that L_x/L_y is of the order of 3 or more. Substituting Eqs. (43) and (41) into Eqs. (38) and (39) and using subsonic data for the velocity fluctuations in the boundary layer, we obtain the results shown in Fig. 5 for the mean square velocity fluctuations in the x direction: Eqs. (45) and (43), when coupled to the subsonic data, yield $L_x/L_y \approx 3.5$ and $L_z/L_y \approx 1.5$ in the boundary layer at the end of the body. The streamline locations correspond to the approximate location of the peak fluctuations. It appears from Fig. 5 that by introducing a degree of anisotropy into the boundary-layer turbulence, results for the decay of the peak velocity fluctuations consistent with observed data can be obtained. However, it should be

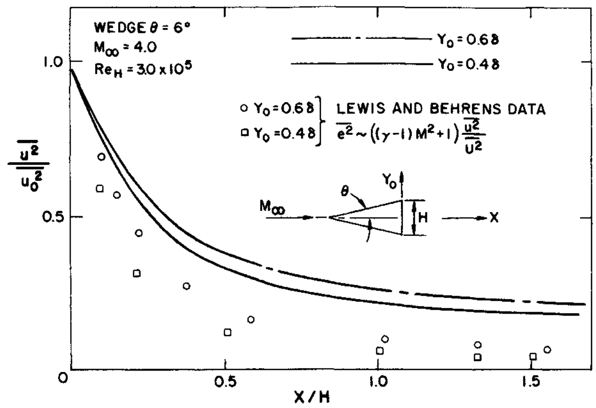

Fig. 5 Results for axial mean square velocity fluctuation decay with an anisotropic boundary-layer turbulence spectrum, and data of Lewis and Behrens interpreted in terms of velocity fluctuations.

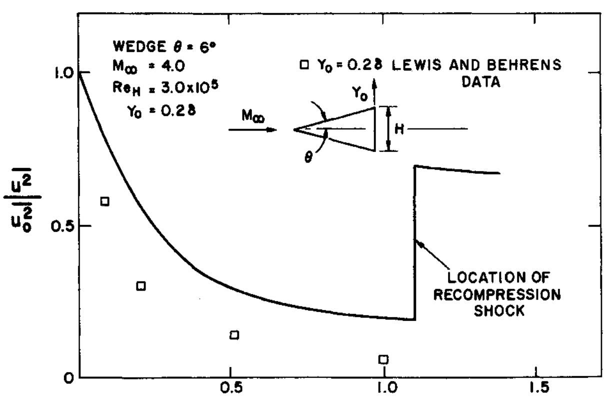

Fig. 6 Results for axial mean square velocity fluctuation decay when $y_0 = 0.2\delta$ with an anisotropic boundary-layer turbulence spectrum, and data of Lewis and Behrens interpreted in terms of velocity fluctuations.

‡ Demetriades[26] has pointed out that this relation results from assuming negligible stagnation temperature fluctuations in the supersonic expansion, and this assumption may not be valid in supersonic turbulence. In addition, the calculated voltage sensitivity coefficients in Ref. 21 may not represent typical measured values for known experimental conditions. When an indirect correction to the assumption of zero temperature fluctuations is introduced and measured voltage sensitivity coefficients for a typical experiment are utilized,[26] the final level of the velocity fluctuations inferred from Lewis and Behrens' data is approximately a factor of two higher than presented in Figs. 4–6.

emphasized that without more information about the integral scales and velocity fluctuations in a supersonic turbulent boundary layer, these results give only a qualitative indication of the decay in the expansion fan. In Fig. 6, the results obtained when the theory is applied to a streamline which is initially closer to the wall ($y_0 = 0.2\delta$) are presented along with the data as it would be interpreted if the peak fluctuations originated on this streamline. It is interesting to note the immediate intensification of the turbulence as this streamline passes through the recompression shock at $x/H = 1.1$ Unfortunately, in the region where the turbulent remnant passes through the recompression shock, there exists no useful data that can be compared with the theoretically predicted intensification.

In conclusion, the linearized analysis properly represents the decay of supersonic boundary-layer turbulence in the near wake expansion of a slender body with a low-turbulence level. It has been shown that the decay is not caused by dissipation, but results from a contraction of a distribution of vorticity filaments by the mean flow; quantitative predictions of the decay of the mean square velocity fluctuations depend upon a proper choice of the boundary-layer turbulent energy spectrum and knowledge of the three components of the velocity fluctuations in a supersonic turbulent boundary layer.

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